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Quantum Corrections for a Cosmological String Solution*

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ABSTRACT

We investigate quantum corrections for a cosmological solution of the string effective action. Starting point is a classical solution containing an antisymmetric tensor field, a dilaton and a modulus field which has singularities in the scalar fields. As a first step we quantize the scalar fields near the singularity with the result that the singularities disappear and that in general non-perturbative quantum corrections form a potential in the scalar fields.

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1. Introduction

In two dimension (2-D) the dilaton gravity could be formulated as a quantum theory in the last years.[1,2,3,4,5] This opens the possibility to quantize higher dimensional theories near regions where a 2-D part factorizes. As a first step one can quantize only the 2-D part and leave the dynamical fields living in the other dimensions as a classical background. This procedure is especially motivated if the 2-D part contains singularities and therefore quantum corrections are expected to become important whereas the other part behaves smooth. In this paper we are going to describe one example with this property. It is a solution of the 5-D string effective action

$$S^{(5)} = \int d^5x \sqrt{G} e^{-2\psi} \left(R + 4(\partial\psi)^2 - \frac{1}{12} H^2 \right) \tag{1}$$

where ψ is the dilaton field and $H_{\mu\nu\lambda} = \partial_{[\mu} B_{\nu\lambda]}$ is the torsion corresponding to the antisymmetric tensor field $B_{\mu\nu}$. The 5-D metric in our example has the following form

$$ds^2 = \rho^2 dw^2 - e^{\lambda} dt^2 + t^2 d\Omega_k^2 = \rho^2 dw^2 + \tilde{G}_{\mu\nu} dx^{\mu} dx^{\nu} \tag{2}$$

where $d\Omega_k^2$ is the volume measure corresponding to a 3-D space with constant curvature k (1, 0, -1). In section 2 we discuss the (classical) solution in detail. It turns out that the functions ρ and λ depend only on t and that the dilaton and torsion are independent of the fifth coordinate y. So, it is possible to reduce the 5-D theory (1) down to a 4-D string theory

$$S^{(5)} \to S^{(4)} = \int d^4x \sqrt{\tilde{G}} e^{-2\phi} \left(\tilde{R} + 4(\partial\phi)^2 - (\frac{\partial\rho}{\rho})^2 - \frac{1}{12} H^2 \right) \tag{3}$$

where $\phi = \psi - \frac{1}{2}\log\rho$ is the 4-D dilaton field and ρ is the modulus field. After this Kaluza–Klein reduction we end up with a 4-D Friedman–Robertson–Walker (FRW) metric with spatial curvature k. This 5-D Kaluza–Klein approach in the string theory corresponds to a compactification where not all other dimensions[c] are compactified on a torus with constant radii. Instead, one of these interior dimensions has a non-constant compactification radius ρ.

In the literature there are some cosmological Kaluza-Klein solutions discussed. Similar to this approach Matzner, Mezzacappa and Wiltshire[6] have used the way through a 5-D theory to find new classical solutions for the Einstein gravity in 4-D. For the string effective action Copeland, Lahiri and Wands[7] were able to find a solution even for higher dimensional interior space. The qualitative behavior is similar to the classical solution we are going to describe in the next section. In the third section we are going to discuss the quantized theory after a s-wave reduction near the singularity and, finally, we summarize our results.

[c]We are working in a critical string theory, i.e. in the critical space time dimension (e.g. 26 for the bosonic case).

2. Classical Theory

The classical solution we are interested in is described in Ref. 8. Let us summarize the main features in this section. The original motivation was to discuss the FRW cosmology in a string Kaluza-Klein approach. If one wants to do this one should first discuss the symmetries. The FRW metrics are the most general metrics describing a spatial homogeneous and isotropic universe. The geometry of the 3-D spatial part is therefore given by a S^3_k which is for $k = +1$ a sphere, for $k = -1$ a pseudo-sphere or for $k = 0$ a flat space. If we now embed this 4-D space in a higher dimensional space time we have to make sure that the higher dimensional metric respects this symmetry. In this case, it means that the 5-D metric has to have the S^3_k spherical symmetry and thus can be written as a Schwarzschild metric, i.e. the 5-D metric has to have the form (2). Furthermore, because we are interested in a 5-D theory which can be reduced to a 4-D FRW solution we have to look for solutions which are independent of the fifth coordinate w. If we start with the spherical case ($k = +1$) and remember that the time in Eq. (2) is the radius of the S^3 we have simply to look for a static black hole solution. Then we have to replace the time in the black hole solution by our fifth coordinate, the radius by our time and we have to switch the signature in the metric correspondingly. By that way we have a solution for $k = +1$ which we have finally to generalize to arbitrary k. Following this procedure and starting with the 5-D black hole solution discussed by Strominger and Horowitz[10] [d] we find as general solution

$$\begin{gathered} ds^2 = \frac{-k+\left(\frac{t_+}{t}\right)^2}{1-\left(\frac{t_-}{t}\right)^2} dw^2 - \frac{1}{(-k+(\frac{t_+}{t})^2)(1-(\frac{t_-}{t})^2)} dt^2 + t^2 d\Omega_k^2 \\ e^{-2(\psi-\psi_0)} = 1 - \left(\frac{t_-}{t}\right)^2 \qquad , \qquad H = 2t_+t_-\epsilon_{3,k} \end{gathered} \tag{4}$$

where the prefactor in the torsion $2t_-t_+ = Q_M$ defines a magnetic charge and $\epsilon_{3,k}$ is the volume form corresponding to $d\Omega_k$, i.e.

$$\epsilon_{3,k} = \left(\frac{\sin\sqrt{k}\chi}{\sqrt{k}}\right)^2 \sin\theta \, d\chi \wedge d\theta \wedge d\varphi \ .$$

After the dimensional reduction we find for the 4-D metric and dilaton

$$\tilde{ds}^2 = -\frac{dt^2}{\left(-k+\left(\frac{t_+}{t}\right)^2\right)\left(1-\left(\frac{t_-}{t}\right)^2\right)} + t^2 d\Omega_k^2 \quad , \quad e^{-2\phi} \sim \sqrt{\left(1-\left(\frac{t_-}{t}\right)^2\right)\left(-k+\left(\frac{t_+}{t}\right)^2\right)} \ . \tag{5}$$

Obviously, this solution is not well defined for arbitrary t. In order to ensure a real dilaton and the right signature in the metric we have instead to restrict the allowed

[d] This solution was first found by Gibbons and Maeda[11] for the higher dimensional case.

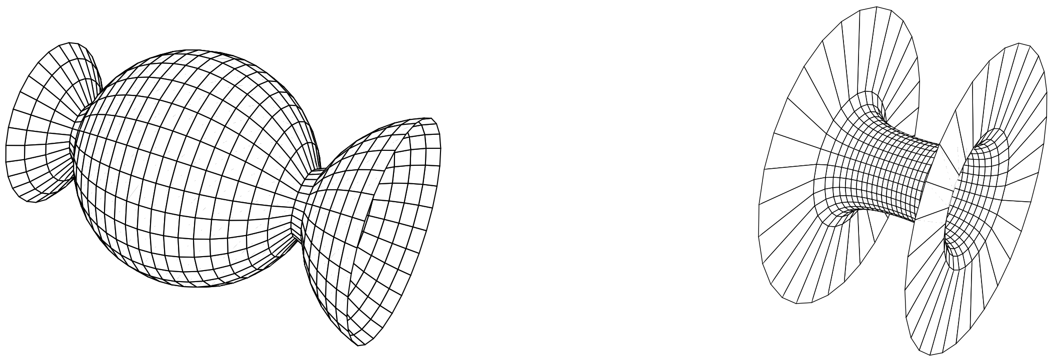

Figure 1: In (a) we have plotted the closed oscillating solution for $k = 1$; (b) is the wormhole solution for $k = -1$.

t region to

$$\left(-k + \left(\frac{t_+}{t}\right)^2\right)\left(1 - \left(\frac{t_-}{t}\right)^2\right) > 0. \tag{6}$$

However, this restriction does not mean that the lifetime is finite, but since t defines the radius of the S_k^3, it means that the spatial extension is bounded. Especially, we have a lower bound for all k as long as t_- is non-vanishing, i.e.,as long as the magnetic charge is non-vanishing (let us assume that, without any restriction, $t_- < t_+$). Since for FRW metrics singularities correspond to zeros or singularities in the world radius we see, that a non-vanishing magnetic charge ($t_- \neq 0$) prevents the universe from forming a singularity. This becomes clear if we transform the metric in the conformal time

$$\tilde{ds}^2 = \left\{t_-^2 + (t_+^2 - kt_-^2)\left(\frac{\sin\sqrt{k}\eta}{\sqrt{k}}\right)^2\right\}\left[-d\eta^2 + d\Omega_k^2\right] . \tag{7}$$

This solution oscillates for $k = +1$ between the minimum t_- and the maximum t_+, for $k = -1$ this solution is asymptotically ($\eta \to \infty$) flat and has wormhole at $\eta = 0$. For $k = 0$ we have no flat regions, but again, the solution shrinks until it reaches at $\eta = 0$ the non-vanishing minimum in order to expand then again. Unfortunately, it is not possible to find an analytic expression for the world radius $a(\tau)$ in the standard parameterization of the FRW metric $ds^2 = -d\tau^2 + a^2(\tau)d\Omega_k^2$. We have plotted numerical results in figure 1.

Remarkably, although the 4-D metric is completely smooth the scalar fields contain divergencies. As one can see in Eq. (4) the modulus field ρ is infinite at $t^2 = t_-^2$ (or $\eta = 0$) and shrinks then with the time. This means, at the minimal extension of the world radius, e.g. inside the wormhole, the compactification radius

ρ is infinite, i.e. there is no compactification. With the further time evolution the world radius expands and the compactification radius shrinks, which means that we have a dynamical compactification. This behavior seems to be quite general. The qualitative feature remains intact also for more than one additional dimensions[7]. Similarly, the dilaton goes to $+\infty$ at the extrema of the world radius $t^2 = t_\pm^2$. There, the theory is in the strong coupling region. The consequence of this behavior is that the Einstein metric becomes singular at these points. This metric for which the effective action has the Einstein-Hilbert term as the first part is defined by the field redefinition

$$G^{(E)}_{\mu\nu} = e^{-2\phi}\, \tilde{G}_{\mu\nu} \ . \tag{8}$$

The vanishing Weyl factor is responsible for the singularity. For a better understanding of this singularities let us discuss the 5-D metric. In the conformal time this metric is given by

$$\begin{aligned} ds^2 &= \left(\frac{\sqrt{k}}{\tan\sqrt{k}\eta}\right)^2 dw^2 + \left\{t_-^2 + (t_+^2 - kt_-^2)\left(\frac{\sin\sqrt{k}\eta}{\sqrt{k}}\right)^2\right\}\,[-d\eta^2 + d\Omega_k^2] \\ e^{2(\psi-\psi_0)} &= 1 + \frac{t_-^2}{(t_+^2 - kt_-^2)\left(\frac{\sin\sqrt{k}\eta}{\sqrt{k}}\right)^2} \ . \end{aligned} \tag{9}$$

If we now approach the singularity at ($\sin\sqrt{k}\eta \simeq 0$) we find that the 5-D metric factorizes in a 2-D (w,η) part and a 3-D spherical part

$$ds^2 = \left(\frac{\sqrt{k}}{\tan\sqrt{k}\eta}\right)^2 dw^2 - t_-^2 d\eta^2 \ + \ t_-^2\, d\Omega_k^2 \quad , \quad e^{2\psi} \sim \frac{1}{\left(\frac{\sin\sqrt{k}\eta}{\sqrt{k}}\right)^2} \ . \tag{10}$$

We see, that the singularities are contained in the 2-D part, which is just the known black hole solution[12] and that the 3-D spherical part behaves smooth. Before we turn to quantize this theory let us make one remark here. One can ask whether there is a limit in which the theory goes over in a conformal field theory (CFT). In a previous paper[8] we have shown that this limit is given by $t_+^2 \to kt_-^2$ and a suitable constant shift in the dilaton. Then the 5-D solution becomes (10) for all η. The result is the direct product of the CFT which is behind the black solution (SL(2,R)/U(1) WZW model) and a 3-D parallized space (for $k = +1$ it is the SU(2) WZW model). Because the spherical part does not depend on the time, in this extremal limit the solution becomes static in the string metric. But, nevertheless, the Einstein metric receives a time dependence via the non-constant dilaton.

3. Quantization and Dilaton/Moduli Potential

In this section we investigate the quantum theory near the singularity. In the figure it is just the region of minimal extension, e.g. inside the wormhole of figure 1b. As we pointed out in the introduction we are not going to quantize this theory completely. Instead, as a first step, we quantize the singular 2-D part and leave

the spherical part as a classical background. During this procedure, which is also known as s-wave reduction, we integrate out the spherical degrees of freedom. This is motivated by the assumption that the quantum corrections respect the spherical symmetry. Generally, this is not the case, but practicable as a first approximation. A further motivation comes from the 4-D theory. The quantization of the 2-D metric and dilaton only corresponds there to a quantization of the scalar fields (moduli and dilaton). This means, our approach corresponds to the quasi-classical approximation

$$R^{(E)}_{\mu\nu} - \frac{1}{2} R^{(E)} G^{(E)}_{\mu\nu} = < T^{(\phi,\rho)}_{\mu\nu} > + T^{(H)}_{\mu\nu} \tag{11}$$

where $G^{(E)}_{\mu\nu}$ is the metric in the Einstein frame (8). When quantizing this theory we are especially interested in what happens with the singularity and whether quantum corrections can form a dilaton/moduli potential.

After the s-wave reduction we get

$$S^{(5)} \to S^{(2)} = \int d^2 z \sqrt{g} e^{-2\phi} \left(R^{(2)} + 4(\partial\phi)^2 + \lambda \right) \ . \tag{12}$$

This is the known dilaton gravity with $\lambda = \frac{2}{t_-^2}\left(3k - (\frac{t_+}{t_-})^2\right)$. As a consistency condition we have to ensure that in the classical limit (weak coupling limit, $\phi \to -\infty$) we get the back the classical solution, which is in conformal coordinates given by [12]

$$ds^2 = e^{2\sigma} dz^+ dz^- \qquad , \qquad e^{-2\phi} \sim e^{-2\sigma} = u - \lambda z^+ z^- \tag{13}$$

where u is constant. This solution can be transformed in the 2d (w, η) part of Eq. (10) where $\eta \simeq 0$ corresponds to $u \simeq \lambda z^+ z^-$.

We are now following the procedure of de Alwis[5]. Choosing the conformal gauge

$$g_{ab} = e^{2\sigma} \hat{g}_{ab} \tag{14}$$

we can write (12) as a general 2d σ model

$$S^{(2)} = -\int d^2 z \sqrt{\hat{g}} \left[\hat{g}^{ab} \partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X) + \hat{R}\, \Phi(X) + T(X) \right] \tag{15}$$

with: $X^\mu = \{\phi, \sigma\}$. Thus, the quantization of the dilaton gravity is reduced to the quantization of a 2d σ model with the target space spanned by ϕ and σ. We can write the metric $G_{\mu\nu}$ as

$$dS^2 = -4e^{-2\phi}[1 + h(\phi)] d\phi^2 + 4e^{-2\phi}[1 + \bar{h}(\phi)] d\sigma d\phi + \kappa d\sigma^2 \tag{16}$$

where h and $\hat{h}$ are model dependent functions of ϕ or X^1, which contain the quantum corrections. For $h = \bar{h} = 0$ we have the CGHS model [1]; for $2h = \bar{h} = -e^{2\phi}$ the model discussed by Strominger[3]; $h = 0$ and $\bar{h} = -\frac{\kappa}{4} e^{2\phi}$ describes the RST model [4]. The parameter $\kappa = \frac{24-N}{6}$ originates from the definition of the functional integration

measure and N corresponds to additional conformal matter. In order to get a flat metric $G_{\mu\nu}$ we introduce as next step new target space coordinates

$$
\begin{aligned}
x &= \tfrac{2}{\sqrt{\kappa}} \int d\phi e^{-2\phi} \sqrt{(1+\bar{h})^2 + \kappa e^{2\phi}(1+h)} \\
y &= \sqrt{\kappa}\sigma - \tfrac{1}{\kappa} e^{-2\phi} + \tfrac{2}{\kappa} \int d\phi e^{-2\phi} \bar{h} \ .
\end{aligned}
\tag{17}
$$

After this we end up with the model

$$
S^{(2)} = \int d^2 z \sqrt{\hat{g}} \left[(\partial x)^2 - (\partial y)^2 + \hat{R}\, \Phi(x,y) + T(x,y) \right] \ . \tag{18}
$$

However, the function $\Phi(x,y)$ and $T(x,y)$ are not arbitrary. The requirement of independence of the reference metric $\hat{g}_{ab}$ has the consequence that the 2-D theory has to be conformal invariant. The simplest choice is to take a linear dilaton Φ and a exponential tachyon T

$$
\Phi = \sqrt{\kappa} y \qquad , \qquad T = \lambda e^{\frac{2}{\sqrt{\kappa}}(x-y)} \ . \tag{19}
$$

With this choice we have a well defined 2-D quantum theory (mathematically the same as the non-critical string theory in one dimension). Now, one defines the quantum theory in terms of these x and y variables and regards Eq. (12) as the classical limit. In order to investigate the influence on the classical solution we have to discuss the equation of motion for x and y ($\hat{R}^{(2)} = 0$)

$$
\partial^2 x = \lambda \frac{2}{\sqrt{\kappa}} e^{-\frac{2}{\sqrt{\kappa}}(x-y)} \qquad , \qquad \partial^2 y = \partial^2 x \ . \tag{20}
$$

Solving these equations we have to restrict ourselves to solutions that reproduce in the classical limit the black hole solution (13). Therefore, we are interested in a solution depending on the product $z^+ z^-$ only and find

$$
x = y = \frac{1}{\sqrt{\kappa}} \left(u - \lambda z^+ z^- \right) \tag{21}
$$

($u = const.$). Using the transformation (17) we can express this solution by ϕ and σ. In doing so we have to fix the up to now arbitrary functions $h(\phi)$ and $\bar{h}(\phi)$. Let us discuss the parameterization suggested by de Alwis: $h = 0$, $\bar{h} = -\frac{1}{2}\kappa e^{2\phi}$. This choice is motivated by the fact that for all values of ϕ and σ the transformation (17) is non-singular and secondly that the range of x and y goes from $-\infty$ to $+\infty$ if ϕ and σ do so. For x and y one gets

$$
\begin{aligned}
x &= \tfrac{1}{\sqrt{4\kappa}} \left(-\sqrt{\kappa^2 + 4e^{-4\phi}} + \sqrt{\kappa} \text{arcsinh} \tfrac{\kappa}{2} e^{2\phi} \right) \\
y &= \sqrt{\kappa} \left(\sigma - \tfrac{1}{\kappa} e^{-2\phi} - \phi \right)
\end{aligned}
\tag{22}
$$

In terms of Eqs. (21) one finds in the weak coupling limit ($e^{2\phi} \ll 1$) the desired classical solution (13)

$$e^{-2\phi} = u - \lambda z^+ z^- \qquad , \qquad \sigma = \phi \ . \tag{23}$$

But our original singularity appeared in the strong coupling region (see Eq. (10)). In this limit ($e^{2\phi} \gg 1$) we obtain

$$\phi = -\frac{1}{\kappa}(u - \lambda z^+ z^-) \qquad , \qquad \sigma = \frac{1}{\kappa} e^{-2\phi} \ . \tag{24}$$

Therefore, after incorporation of quantum corrections ($\sim \mathcal{O}(e^{2\phi})$) the black hole solution becomes smooth also in the strong coupling region[5]. Note, that in the dilaton gravity a singularity in the metric has to be accompanied by a singularity in the dilaton, i.e. singularities can only appear in the strong or weak coupling region. For the other models the picture is qualitatively the same[9], i.e., after quantizing the theory the singularities disappear. This has immediately the consequence that the Einstein metric in Eq. (8) becomes smooth, too. Thus, the world radius in both frames has a lower bound.

One can now ask, what is the influence of this quantization procedure for the further evolution of the universe? For the derivation of our results it was crucial that the solution decouples in a 2-D (dilaton gravity) part and a 3-D spherical part. This is valid only if one considers the theory, e.g.,inside the wormhole of fig. 1b. Extending this procedure to the region away from the wormhole seems to be difficult. But nevertheless, quantum corrections inside the wormhole can form a dilaton potential which could be a source of an inflationary period in later times. A dilaton potential in our original action (1) or (3) corresponds to an additional tachyon contribution in the 2-D action (12). The tachyon we have discussed so far is only *one* possibility. Although this solution has the advantage that the renormalization group β functions vanish, and thus, yielding a finite 2-D quantum field theory there are other possible tachyon fields. The most general tachyon field is a combination of the solutions of the Weyl invariance condition, which are given by

$$\begin{aligned} &\Phi(x,y) = ax + by && \text{with} \quad a^2 - b^2 = -\kappa \quad , \\ &T(x,y) \sim e^{\alpha x + \beta y} && \text{with} \quad \tfrac{1}{2}(\alpha^2 - \beta^2) - a\alpha + b\beta - 2 = 0 \ . \end{aligned} \tag{25}$$

In order to get the right classical limit we set furthermore $a = 0$. But there is also another parameterization for the tachyon.[e] Using the mass shell condition we can replace α or β and then we can expand the tachyon field in powers of the remaining α or β. Since the tachyon β function is a linear equation in T every term in this expansion fulfills the β equation, too. In the language of 2d conformal field theories, this means that every term is an allowed (1,1) operator. This procedure

[e]For throwing our attention on this possibility we are grateful to S. Förste

for constructing of new vertex operators was described by Kawai and Nakayama[13]. After this procedure we find an infinite set of vertex operators or tachyon fields which are parameterized by two integers m and n. If we restrict ourselves on $\kappa = \frac{24-N}{6} = 4$ (i.e. $N = 0$) these additional terms are

$$T_2^{(n)} = (y-x)^n \, e^{2x} \quad , \quad T_3^{(m)} = (x \pm y)^m \, e^{2y} \ . \tag{26}$$

Instead of Eq. (25) we have now as general tachyon field $T(x,y)$

$$T(x,y) = \lambda e^{\frac{2}{\sqrt{\kappa}}(x-y)} + \sum_{(n,m)} (\mu_2^n T_2^{(n)} + \mu_3^m T_3^{(m)}) \tag{27}$$

where the function x and y are given by the Eq. (22) (the term $T_2^{(0)}$ was already discussed in Refs. 5 and 14). A remarkable property of these terms is, that they have in the classical limit ($\phi \to -\infty$) the typical non-perturbative structure

$$T_{2,3} \sim e^{-\frac{1}{2}e^{-2\phi}} \sim e^{-\frac{1}{(2g_s^2)}} \tag{28}$$

where $g_s = e^{\phi}$ is the string coupling constant and we have used that $h, \bar{h} \sim \mathcal{O}(e^{2\phi})$ (because they are quantum corrections). On the other side, in the strong coupling region ($\phi \to \infty$) we have

$$T_2 \sim e^{4\phi} \to \infty \quad , \quad T_3 \sim e^{-2\phi} \to 0 \tag{29}$$

Therefore, these terms vanish very rapidly in the weak coupling (classical) region and become important in the strong coupling region. Furthermore, since x and y are functions of the dilaton and moduli field[f] these additional tachyon terms represent a dilaton/moduli potential created by non-perturbative quantum corrections in the strong coupling region. It remains an open question whether this potential can yield sufficient inflation. In order to discuss this question one has to transform the theory back to the 4-D Einstein frame and has to show that the resulting potential has a flat direction which can then yield to extended inflation[15]. Probably, this is possible for a suitable choice of the constants $\mu_{2,3}^{m,n}$. However, normally in discussing of non-perturbative corrections one imposes further string symmetries to restrict the possible contributions[16] and it deserves further investigations to show that this will not destroy a flat direction.

4. Conclusions

The aim of this paper was to show how quantum corrections modify a cosmological solution of the string effective action. The classical solution was obtained by a 5-D Kaluza-Klein approach, with an antisymmetric tensor and dilaton field

[f] In order to get the moduli field ρ from the Weyl field σ one has to go from the conformal gauge in (14) to the Schwarzschild gauge ($ds^2 = \rho^2 dw^2 - d\eta^2$).

as matter part. The 4-D FRW metric in the string frame is completely smooth as long as the torsion is non-vanishing. The corresponding magnetic charge prevents the universe from collapsing. The solution is oscillating for $k = +1$ and a time-like wormhole for $k = -1$. But nevertheless, the scalar fields, the dilaton and the moduli, are singular at the points of minimal and maximal extension of the universe. The moduli field or compactification radius, e.g., is infinite inside the wormhole and shrinks then with the time (dynamical compactification). For the quantization of this classical theory it was crucial that near the singularities the 5-D theory factorize in a singular 2-D part and a smooth (spherical) 3-D part. As a first step, after a s-wave reduction we have quantized the singular part only. From the 4-D point of view this partial quantization is a quasi-classical approximation, where we quantized the scalar matter part (dilaton and moduli) only and leave the metric and antisymmetric tensor as a classical background. As result of this procedure the singularities in the scalar fields disappeared and a dilaton/moduli potential can be formed. The smoothness, especially of the dilaton fields, had the consequence that the 4-D Einstein metric became nonsingular, too. The potential, on the other side, was created by non-perturbative quantum corrections. We have discussed in principle what type of potential is possible from the quantization of the scalar fields. But before one starts to discuss an inflationary period driven by this potential one has first to restrict this potential by imposing of other string symmetries which remains a question of further investigations.

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